Analysis and Design of Dynamic Biped Walking with a 14-DOF Model

Zeki Y. BAYRAKTAROGLU and K. Oytun YAPICI

Istanbul Technical University, Mechanical Eng. Dept., MA1 Gumussuyu 34437, Istanbul, Turkey

{zeki.bayraktaroglu , yapici}@itu.edu.tr

Abstract

This paper presents an analysis of biped walking whose dynamics has a great tendency for instability. A 14-DOF biped model has been used in dynamic walking analysis. Open and closed loop controllers have been designed and applied in simulations. The general control scheme has been based on the ZMP control with linear inverted pendulum modeling. A flywheels based approach has been proposed and applied in simulations in order to achieve the closed-loop control of the system with respect to disturbing dynamic effects occurring in dynamic walking. The principle of the flywheels approach is based on the direct compensation of the ZMP disturbances at the upper body level. Dynamic walking over linear and curvilinear trajectories, lateral and backward walking and walking up and down the stairs have been simulated as the fundamental behaviors in biped locomotion. Forward walking speeds of 6km/h has been observed in simulations. The analysis results presented in this paper have proved to be the basis of the ongoing design of a biped walking robot.

Keywords: Biped locomotion, dynamic walking, ZMP control

1. INTRODUCTION

Human-like biped locomotion happens to be the fundamental behavior of interest in humanoid robotics research. Mobility due to biped walking is the key issue in the developments achieved on humanoid robot design which has been based on earlier research on biped locomotion [1-22]. Original approaches have been proposed since the pioneering works of Vukobratovic and Kato in late 1960s [1-4]. Later publications of Vukobratovic et al. on the subject have given very precious highlights of the state-of-art and future perspectives [5-8]. Once the physical phenomena governing the biped locomotion have been understood, it has been possible to develop dedicated control strategies necessary for satisfactory walking performances. Some of the recently presented biped robots among the others are described in references [9-16].

The stability of equilibrium is the key problem in the analysis and modeling of biped walking mechanisms. One can distinguish the so-called *static* and *dynamic* walking behaviors in human-like biped locomotion. The static walking is defined as the walking behavior where the static equilibrium is instantaneously maintained during locomotion. Stability of static equilibrium can be guaranteed only with low speed motions of the mobile mechanism. The dynamic walking is characterized by higher speed motions of the mobile mechanism. Dynamic effects occurring in dynamic walking must be considered in modeling and controller design for a stable behavior. Modeling, analysis and design of dynamically walking biped mechanisms have been based on the widely accepted notion of Zero Moment Point proposed by Vukobratovic [5].

This paper presents an integrated approach for the analysis and design of a dynamically walking biped robot. Open- and closed-loop ZMP controls of dynamic biped walking have been simulated in a modular computer environment. Beside the biologically inspired mechanical structure of the biped robot, analogies between natural human walking and the proposed walking behavior have been observed in simulations. Simulation results presented in this paper have founded the basic research in biped locomotion and served as the basis of a biped robot design.

The following section describes the mechanical structure of the proposed biped model as well as the modeling of biped walking behavior. The third section deals with the control approach applied in simulations. Simulation results and a discussion are given respectively in the fourth and fifth sections.

2. BIPED MODEL

2.1. Dynamics

In this work, the forward dynamics of the walking behavior is to be computed in simulations. The inverse dynamics model must be used in the computation of actuator input torque in order to obtain high speed locomotion, i.e. dynamic walking behavior. Dynamics of an open chain mechanism is given in closed-form as follows:

$$M(\theta)\ddot{\theta} + V(\theta,\dot{\theta}) + G(\theta) + F(\theta,\dot{\theta}) = \tau \qquad (1)$$

where n being the number of DOF, $M(\theta)$ is the $n \times n$ inertia matrix, $V(\theta,\dot{\theta})$ the $n \times 1$ vector of coriolis and centrifugal forces, $G(\theta)$ the $n \times 1$ vector of gravitational forces, and $F(\theta,\dot{\theta})$ the $n \times 1$ vector of friction forces.

The inverse dynamics algorithms used in the analysis and simulation of multi-body systems have been developed in iterative form suitable for programming [23]. The Paul-Luh-Walker algorithm [24] chosen for simulations is a force-based algorithm using the classical Newton-Euler formulation. One advantage of the Paul-Luh-Walker algorithm is the execution of computations in local reference frames attached to the links, which decreases the total number of necessary mathematical operations. In Lagrangian based approaches, one needs to write the expression of the total potential energy with respect to a fix reference frame, which increases the computational load.

2.2. Single and double support phases

In single support phase, the mechanism is in open chain form and the 12-DOF of the legs are independent from each other. With suitable assumptions on friction effects, the mobile mechanism can be considered as a 12-DOF manipulator attached to the ground (Figure 1).

The original Paul-Luh-Walker algorithm being developed for serial manipulators, it has been adapted to the mechanical structure of a biped walking mechanism [25]. The open kinematic chain based on the foot at ground has a tree structure as given in Figure 2. Two of the branches follows the flywheels on the upper body and the other one the swinging leg. This tree structure has been considered in the adaptation of the Paul-Luh-Walker algorithm.

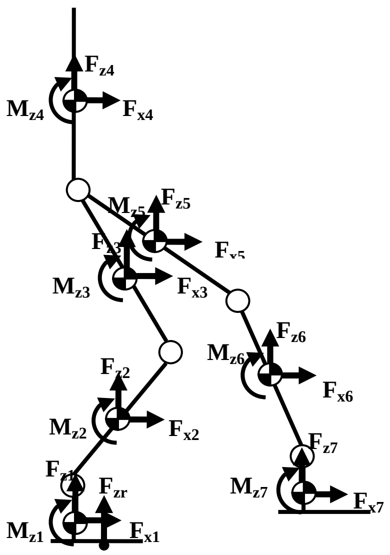


Figure 1: Single support phase with a swinging leg

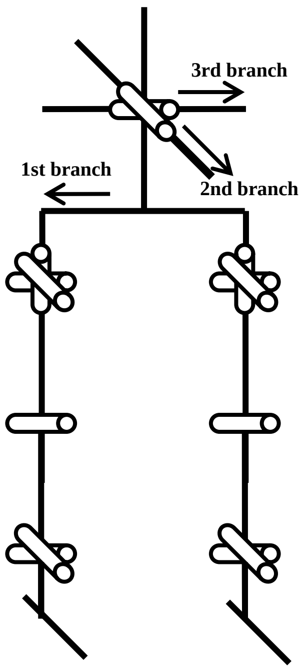


Figure 2: Tree structure

In double-support phase, the second foot being also in contact with the ground, the resulting reaction forces and torque have been added into the algorithm [26]. In this way, these reactions considered as external effects have been compensated by joint actuators along the legs. In simulations, ground reactions have been measured at the 4 corners of the rectangular foot base and reduced to a reference point on the foot.

3. WALKING MODELING AND CONTROL

In biped mechanisms, the 6 DOF at the foot-ground interaction are not directly controllable. The so-called Zero Moment Point (ZMP) of the system must remain in a region on the ground covered by the feet for the stability of the walking, i.e. the support polygon (Figure 3). Foot-ground contact can be maintained as required with the controllers based on the ZMP definition [5, 10].

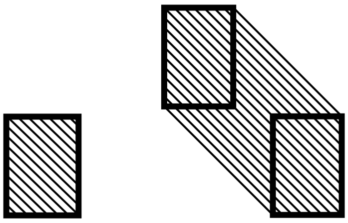


Figure 3: Support polygons for single and double support phases

3.1. ZMP and linear inverted pendulum modeling

Main forces and torque acting on the mechanism due to walking are the gravitational and inertial forces with the ground reactions. The ZMP is defined as the point on the ground plane where the sum of all torque with respect to this point equals zero. The position of the ZMP on the ground plane is written as follows [27, 28]:

$$X_{zmp} = \frac{\sum(F_{ix} z_i - F_{iz} x_i) + \sum M_{iy}}{\sum F_{iz}} \quad , \quad Y_{zmp} = \frac{\sum(F_{iy} z_i - F_{iz} y_i) - \sum M_{ix}}{\sum F_{iz}} \qquad (2)$$

where F and M represent respectively the total forces and torque reduced to the Centre of Gravity

(CoG) of the links.

Equations (2) show the nonlinear analytic expression of the ZMP. These expressions have been simplified based on the assumption that the entire mechanism can be reduced to a point mass concentrated on the CoG of the system [29-32]. The system can then be approximated by a linear inverted pendulum model as shown in Figure 4. If, in addition, the height of the CoG from the ground is assumed to remain constant, then the forces and torque acting on the system will consist of the gravitational force and the force due to the forward acceleration on the CoG.

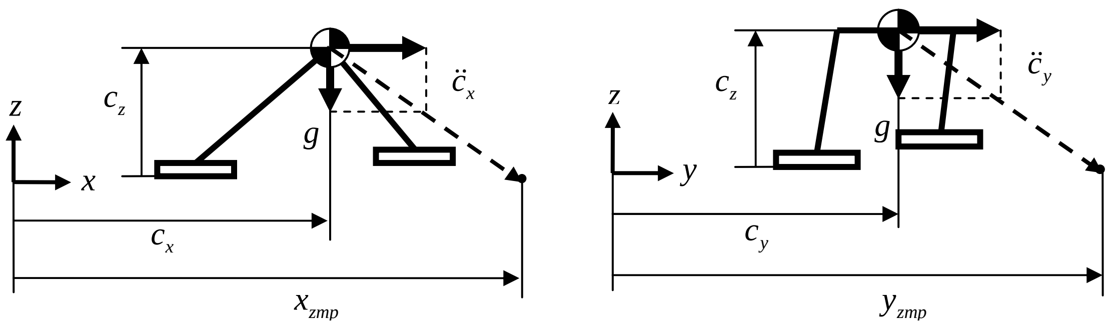


Figure 4: Linear inverted pendulum modeling

Considering the motions on both x and y directions, the ZMP expressions (2) can be simplified as follows:

$$c_x - \frac{c_z}{g}\ddot{c}_x = x_{zmp} \quad , \quad c_y - \frac{c_z}{g}\ddot{c}_y = y_{zmp} \tag{3}$$

This so-called linear inverted pendulum modeling will simplify the solution of the differential equations and therefore the computation of the reference CoG trajectories with respect to the desired trajectories of the ZMP.

If the walking speed is nearly constant then the accelerations are negligible and the equations (3) reduce to as follows [33]: $\ddot{c}_x \approx 0 \, , \ddot{c}_y \approx 0 \quad \Rightarrow \quad c_x \approx x_{zmp} \, , c_y \approx y_{zmp}$. The assumption of negligible accelerations can be satisfied in sufficiently low speeds and relatively longer double-support phases, i.e. static walking.

3.2. Reference trajectories

The walking control scheme requires the reference trajectories of the feet and CoG as inputs. The feet must be transferred between successive positions on the ground while respecting the continuity constraints on their velocity and acceleration values at contact instants with the ground [34]. Continuity up to the 2^{nd} order and ease of adjustability have been taken into account in the search of suitable trajectories. Simulation based studies have shown that velocity references instead of position references provide smoother feet trajectories. This observation is in agreement with the natural human locomotion over flat surfaces during which the walking velocity is controlled but not the feet positions. A biped walking with constraints on the feet positions in a trajectory will clearly disturb the periodic and smooth character of the motion. Therefore the velocity control of biped walking will result with smoother motion than the position control.

Figure 5 shows four different foot trajectory among the others used in simulations. The reference profiles of the feet for forward walking can be expressed by unique functions while the reference profiles for stair climbing are rather given by piecewise functions. The piecewise structure of the reference profiles over the z axis provides the possibility of adjustment of the step heights independently from each other, which is necessary in stair climbing.

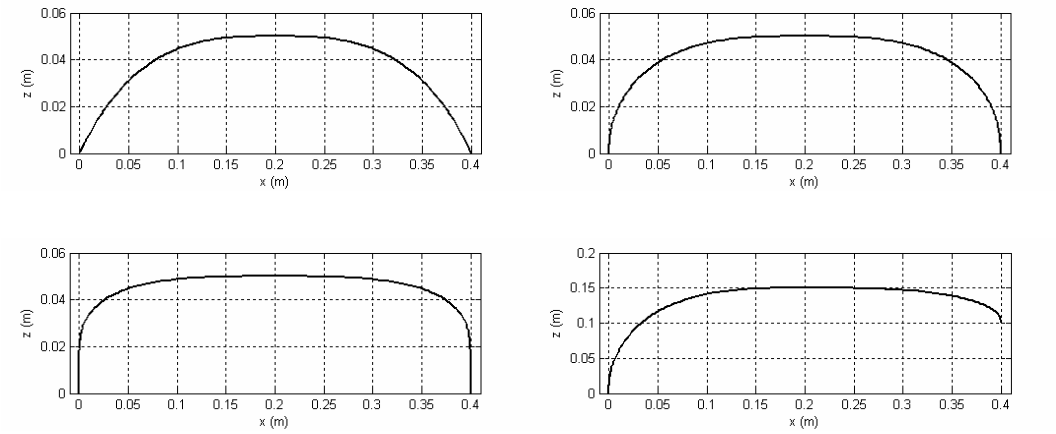


Figure 5: Reference foot trajectories for walking over horizontal ground and stair climbing

The reference CoG trajectories in dynamic walking can be obtained by solving the equations (3). Analytical and numerical solution methods as well as the solution through servo-control approach have been considered. The support polygon during walking is defined as the area on the ground given by the interaction surface between the foot and ground. The walking stability is directly related to the ZMP position with respect to the support polygon. Figure 12 shows the stability margins as a function of the ZMP position in single support phase.

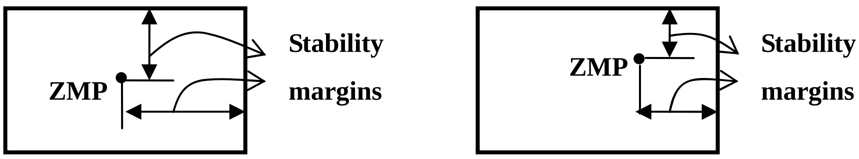


Figure 12: The ZMP positions and stability margins

3.3. Servo-control around the linear inverted pendulum model

Simulation based studies have shown that the solution of the equations (3) through the servo-control approach [35, 36] instead of analytical [37] and numerical [38] methods has provided relatively smoother trajectories and therefore smoother dynamic walking performances. If the jerk u_x of the CoG, i.e. the time-derivative of the CoG acceleration in forward direction is defined as a new state variable, then the equations (3) can be written in a suitable state-space form as follows:

$$\begin{pmatrix} \dot{x} \\ \ddot{x} \\ \dddot{x} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} x \\ \dot{x} \\ \ddot{x} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \cdot u_x \quad , \quad \begin{pmatrix} x_{zmp} \\ c_x \end{pmatrix} = \begin{pmatrix} 1 & 0 & -z_c/g \\ 1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} x \\ \dot{x} \\ \ddot{x} \end{pmatrix} \tag{4}$$

Figure 13 shows the block diagram of the servo-control approach used for the resolution of the equations (4).

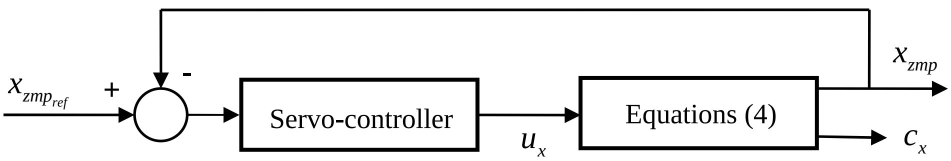


Figure 13: Block diagram of the servo-control around the linear inverted pendulum model

The dynamics described by the above servo-control system is unstable and the CoG reference of the system is in advance in time with respect to the ZMP reference as shown in Figure 14. In these conditions, the servo-control approach can be applied only if the future values of the ZMP reference are known to the servo-controller [39].

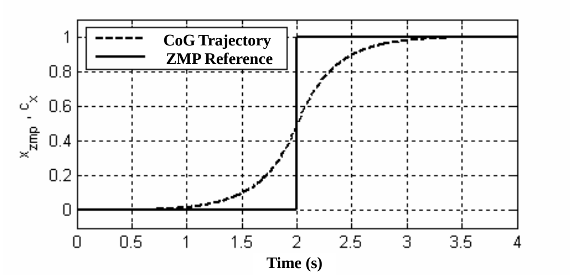


Figure 14: CoG and ZMP reference trajectories

3.4. Model prediction and computed torque control

The required prediction of the ZMP trajectories has been obtained with a preview control approach on the control of the HRP [35]. The study of the prediction problem has shown the analogies between the preview and model predictive control methods [40, 41]. In this work, the well-known model predictive control method has been applied to solve the prediction problem. Continuous-time equations (4) have been transformed in discrete-time domain with the sampling period h as follows:

$$\underline{x}(k+1) = \begin{pmatrix} 1 & h & h^2/2 \\ 0 & 1 & h \\ 0 & 0 & 1 \end{pmatrix} \cdot \underline{x}(k) + \begin{pmatrix} h^3/6 \\ h^2/2 \\ T \end{pmatrix} \cdot u(k) \quad , \quad p_x(k) = \begin{pmatrix} 1 & 0 & \frac{-z_c}{g} \end{pmatrix} \cdot \underline{x}(k) \tag{5}$$

The above discrete-time model and the simulation platform have been used in order to determine the suitable predictive control parameters, i.e. the prediction horizon, control horizon, control interval and total weight. The CoG trajectories obtained with the above defined model predictive controller are given in Figure 15.

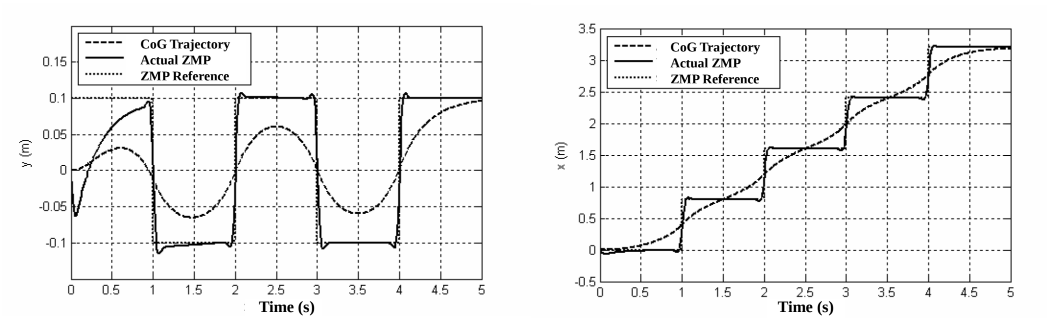


Figure 15: CoG trajectories and ZMP references in x and y directions

A computed torque control scheme (Figure 17) is used in computing the actuator inputs. The proportional and derivative gains of the PD controller have been tuned by trial and error approach.

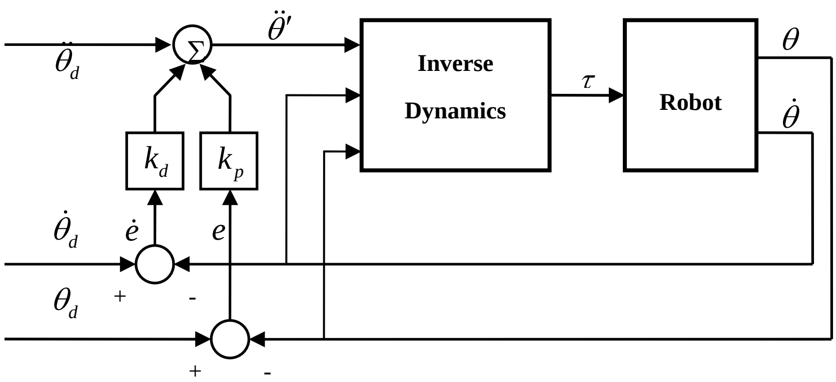


Figure 17: Computed torque control scheme

3.5. Closed-loop control of dynamic walking

We have made a number of assumptions in order to obtain simpler mathematical models to be used in the control loops. Signal noise and any other external perturbations have been neglected. Effects of the unmodeled dynamics occurring especially while walking over curvilinear trajectories and stair climbing can be considered among perturbations due to the walking dynamics. In order to cancel these unwanted effects a closed-loop controller for the ZMP position has been designed and applied in simulations. Two flywheels with orthogonal axes in the horizontal plane have been added to the upper body of the biped model as shown in Figure 18. These flywheels have been modeled as second-order dynamics and used in counter-balancing the torque measured at feet (Figure 19). Flywheel dynamics have also been added in the inverse dynamics mentioned in Section 2.

Figure 18: Flywheels

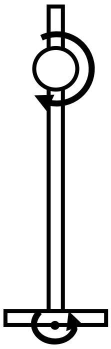


Figure 19: Torque balancing through flywheels

Physically, the problem of biped walking control can be considered as the one of orientation control of the system. Since the ZMP position under the foot is directly related to the configuration of the entire mechanism, walking stability can be achieved only if the orientation of the mechanism can be controlled. The torque measured at feet is directly fed to the flywheels therefore the ZMP position is continuously attracted by the ground region under the feet, i.e. the support polygon. The general block diagram of the control system used in simulations is given in Figure 20.

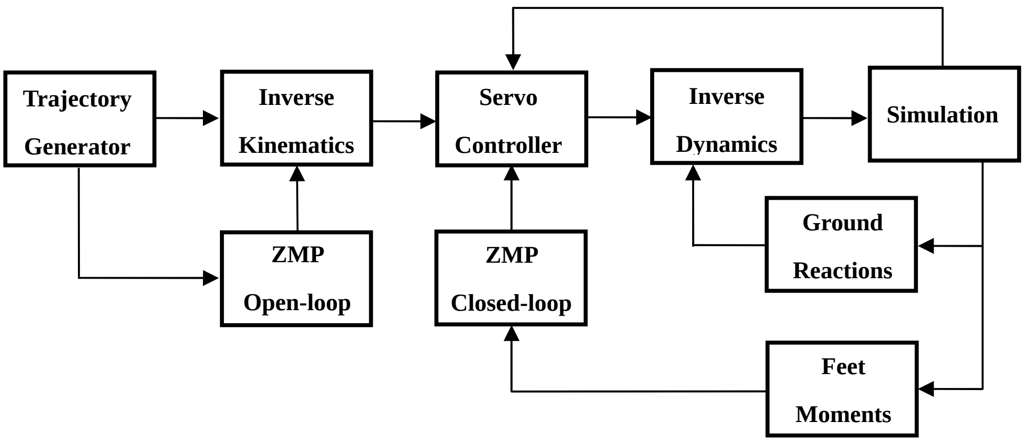


Figure 20: Block diagram of the general control structure

4. SIMULATIONS

Simulations of dynamic walking have been realized in a modular software platform using MSC Adams™ and Matlab™. The constant frequency of communication between these software is determined by the sampling period of 0,0005 seconds used in the computation of the forward dynamics. The sampling period mainly depends on the dynamics caused by fast varying ground reactions. Simulations have been run on a PC with a 2.13 GHz Core 2 Duo processor. It has been observed that MSC Adams™ fully used one core and Matlab™ needed 60% of the capacity of the second core.

Figure 21 shows the simulation environment and 6 images captured during dynamic walking. In the images given in Figure 21, the biped model achieves respectively the following walking behaviors: a) Walking over linear trajectory (Forward walking), b) Stair climbing, c) Walking over a curvilinear trajectory (Turning left), d) Walking down the stairs, e) Lateral walking, and f) Backward walking. The height and weight of the biped model used in simulations are respectively 1390mm and 30kg.

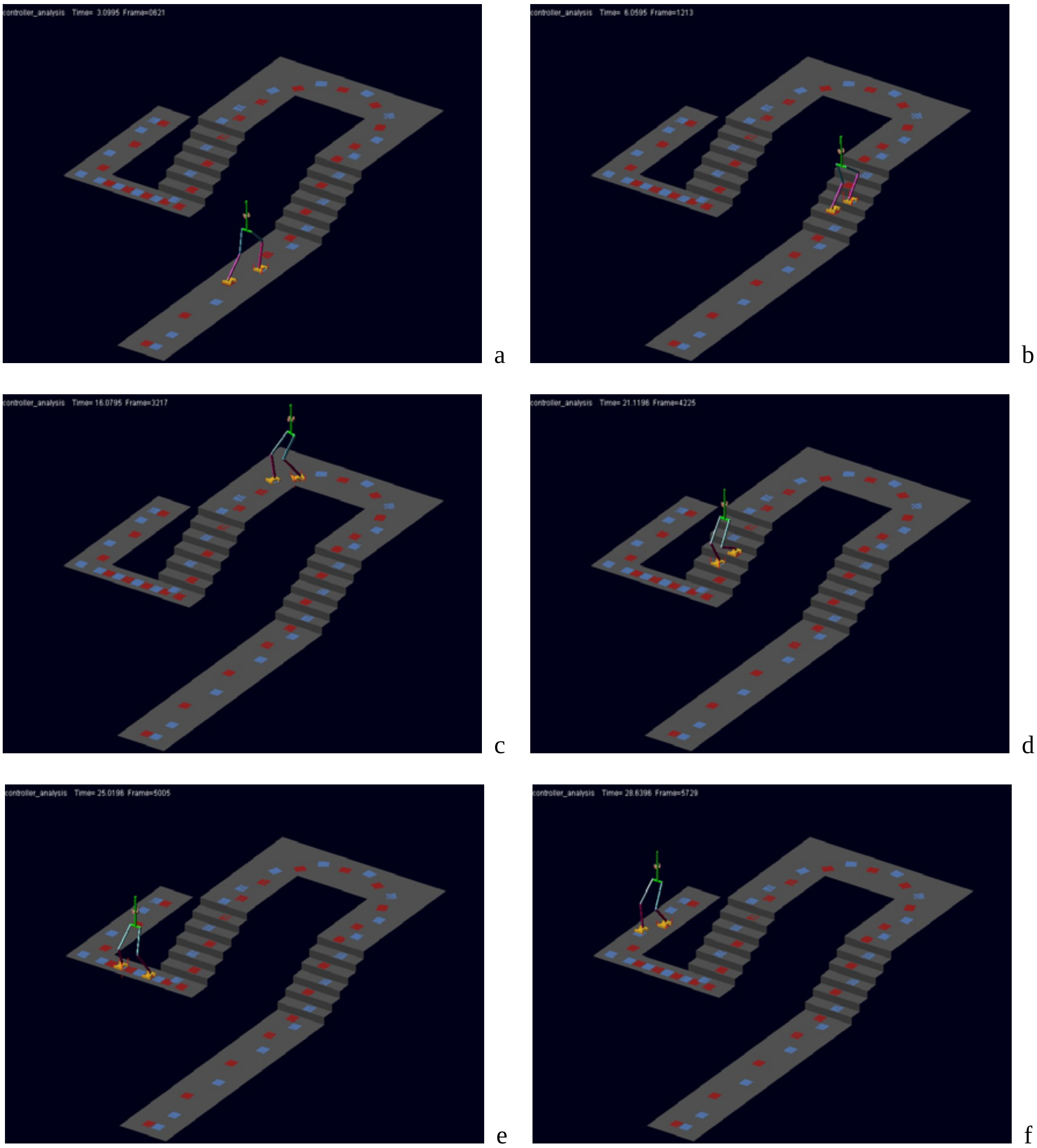


Figure 21: Simulation environment and captured images

4.1. Forward walking over linear trajectories

Forward walking simulations have been realized using several different parameter sets corresponding to various combinations of the step length L and period T. Figure 21 shows examples of the reference and realized trajectories of the ZMP as well as the trajectory of the CoG.

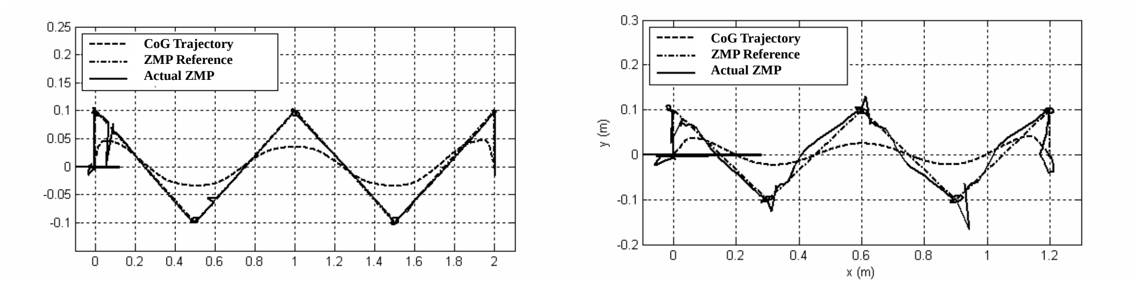


Figure 21: The reference and realized trajectories of the ZMP and the trajectory of the CoG for (a) $T = 0,5s$, $L = 30cm$, (b) $T = 0,75s$, $L = 50cm$

4.2. Stair climbing

15cm height stairs have been used in stair climbing simulations (Figure 22). The CoG of the model has been given constant reference velocities in vertical directions. Discontinuous velocity references have been smoothened by second order low-pass filters. Deviations of the CoG from its reference remain limited as shown in Figure 23.

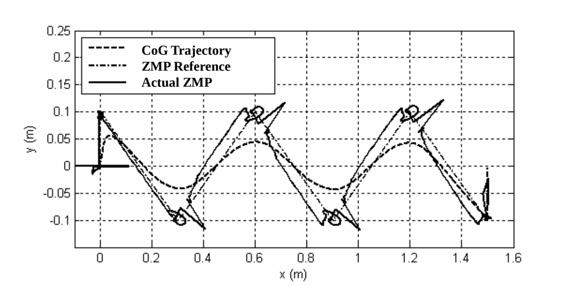


Figure 22: The reference and realized trajectories of the ZMP and the trajectory of the CoG while climbing stairs

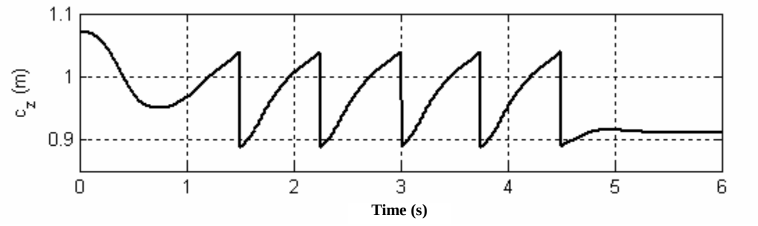


Figure 23: Motion of the CoG over the vertical axis

4.3. Forward walking over curvilinear trajectories (Turning right and left)

Centrifugal forces act on the model while walking over curvilinear trajectories. Compensation of this unwanted dynamic effect has been addressed in some of the biped robots [42]. In the present work, flywheels mounted on the upper body have been used for the closed-loop compensation (Figure 24). Figure 25 shows the CoG trajectory of the mechanism while walking over a curvilinear trajectory.

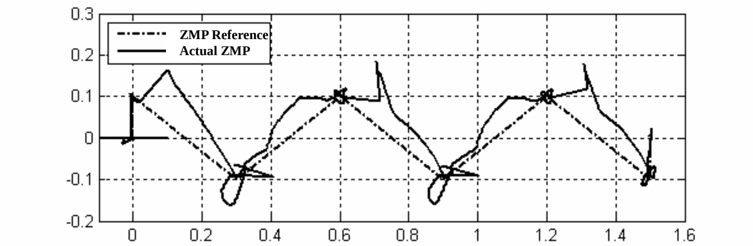


Figure 24: The reference and realized trajectories of the ZMP over a curvilinear trajectory

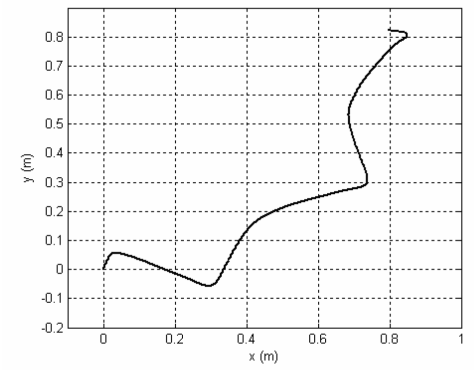
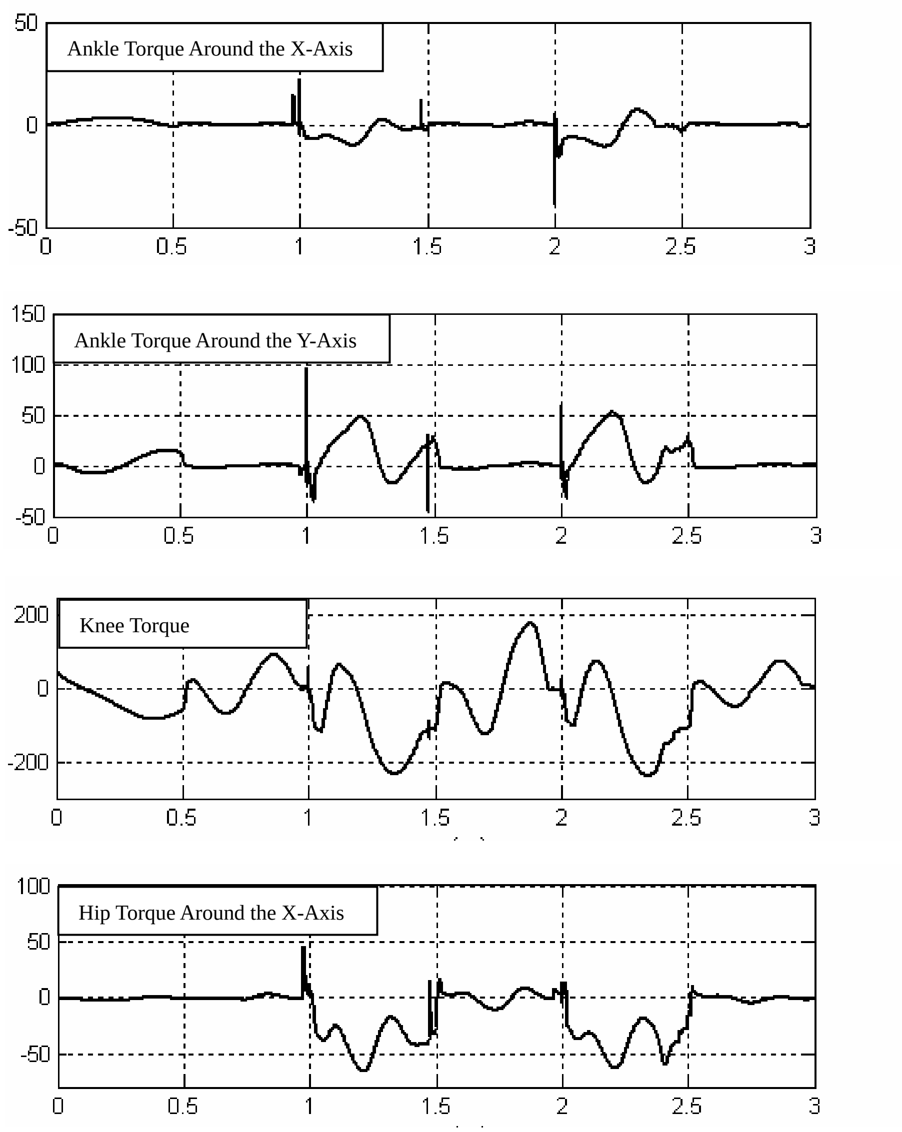


Figure 25: The trajectory of the CoG over a curvilinear trajectory

4.4. Joint torques

Actuator inputs required for the above presented locomotion behaviors have been computed in the controller and applied to the joints in simulations. Joint torques measured at one leg during forward walking at 5 km/h are given an example in Figure 26. Simulation results have shown that the required power at joint levels can be provided with commercially available DC motors.



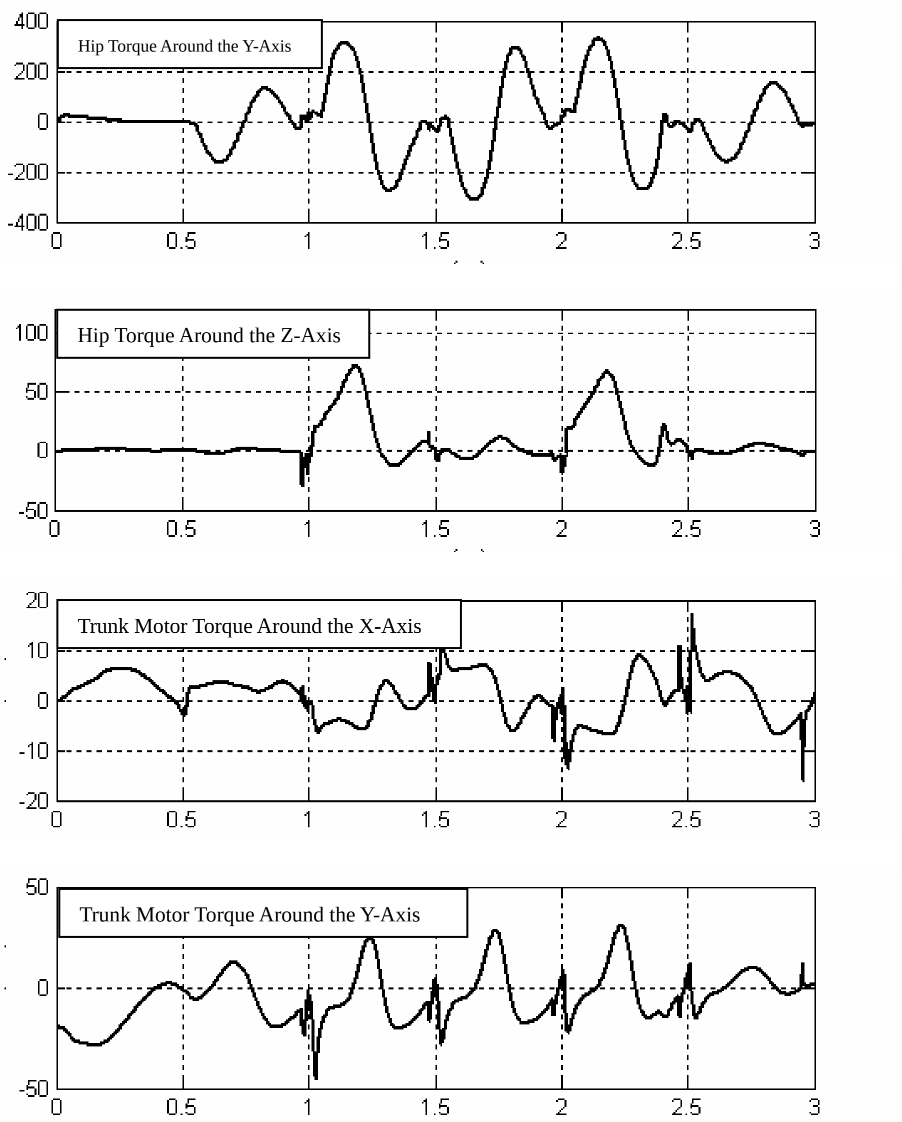


Figure 26: Torques (Nm) versus time (sec) graphics for the actuator inputs in 5 km/h forward walking

5. DISCUSSION

A biped model and related controllers for dynamic walking have been proposed and successfully tested in a modular simulation environment. The mechanical structure of the biped model used in simulations has been biologically inspired, the 6 DOF per leg of the model being also the principal DOF with relatively larger range of angular displacements present in human leg. It has been observed in simulations that velocity references for feet provide smoother walking behavior than position references. Natural human walking over flat surface is also based on velocity control; over unconstrained ground, we do not care about where we put our feet, only the walking speed is controlled. Therefore the simulation results are in agreement with natural human walking, i.e., the biological inspiration source.

Various control approaches present in literature have been studied and a unified scheme has been proposed and tested for the control of dynamic biped walking. Simulation results have shown that walking stability can be achieved with open-loop ZMP controller in forward, lateral and backward walking as well as stair climbing. While walking over curvilinear trajectories, i.e. turning left and right, the robot is instantaneously rotating about an axis perpendicular to the ground. This rotation results with fictitious forces such as Coriolis and centrifugal forces acting on the walking robot. These forces show disturbing effects on the stability of the ZMP and hence the robot. A complementary closed-loop ZMP controller has been used for compensating the disturbing dynamics occurring in walking over curvilinear trajectories. The closed-loop controller helps only the robot reaching higher velocities over curvilinear trajectories and do not have any effect over linear trajectories.

Simulation results have shown that joint torques necessary for dynamic walking can be provided by commercially available electrical actuator and gear combinations. The design of a new biped walking robot has been based on the analysis results presented in this paper.

ACKNOWLEDGEMENT

This work has been supported by the Scientific and Technological Research Council of Turkey, with the TUBITAK Research Project No. 106M340.

REFERENCES

[1] M. Vukobratovic and D. Juricic, A Contribution to the Synthesis of Biped Gait, *in Proc. IFAC Symp. Technical and Biological Problem of Control*, Yerevan, USSR (1968)

[2] I. Kato, S. Ohteru, H. Kobayashi, K. Shirai, and A. Uchiyama, Information-Power Machine with Senses and Limbs, in *Proc. CISM IFToMM Symp. Theory and Practice of Robots and Manipulators*, Udine, Italy, pp.12-24 (1973)

[3] M. Vukobratovic, D. Hristic and Z. Stojiljkovic, Development of Active Anthropomorphic Exoskeletons. *Medical and Biological Engineering*, 12(1) (1974)

[4] A. Takanishi, M. Ishida, Y. Yamazaki and I. Kato, The Realization of Dynamic Walking by the Biped Walking Robot WL-10RD, in *Proc. Proceedings of Int. Conf. Advanced Robotics*, pp.459-466 (1985)

[5] M. Vukobratovic, B. Borovac, D. Surla and D. Stokic, *Biped Locomotion*, Springer-Verlag, Berlin, (1990)

[6] M. Vukobratovic, Beginnings of robotics as a separate discipline of technical sciences and some fundamental results - a personal view, *Robotica*, 20, 223-235 (2002)

[7] M. Vukobratovic and B. Borovac, Zero-Moment Point - Thirty Five Years of its Life, *Int. J. of Humanoid Robotics*, 1(1), 157-173 (2004)

[8] M. Vukobratovic, Humanoid Robotics - Past, Present State, Future, in *Proc. 4th Serbian-Hungarian Joint Symp. on Intelligent Systems*, pp.13-31 (2006)

[9] A. Takanishi, H.O. Lim, M. Tsudaand and I. Kato, Realization of Dynamic Biped Walking Stabilized by Trunk Motion on a Sagittally Uneven Surface, *in Proc. of IEEE Int. Workshop on Intelligent Robots and Systems*, Vol. 1, pp.323-330 (1990)

[10] F. Pfeiffer, K. Loffler, M. Gienger and H. Ulbrich, Sensor and Control Aspects of Biped Robot Johnnie, *Int. J. of Humanoid Robotics* 1(3), 481-496 (2004)

[11] I.W. Park, J.Y. Kim, J. Lee and J.H. Oh, Mechanical Design of Humanoid Robot Platform KHR-3, in *Proc. IEEE/RAS Int. Conf. on Humanoid Robots*, pp.321-326 (2005)

[12] K. Akachi, K. Kaneko, N. Kanehira, S. Ota, G. Miyamori, M. Hirata, S. Kajita and F. Kanehiro, Development of Humanoid Robot HRP-3, in *Proc. IEEE/RAS Int. Conf. Humanoid Robots*, pp.50-55 (2005)

[13] E.V. Cuevas, D. Zaldivar and R. Rojas, Bipedal Robot Description, *Freie Universitat Berlin Institut fur Informatik Technical Report*, B-04-19 (2005)

[14] S. Lohmeier, T. Buschmann, H. Ulbrich and F. Pfeiffer, Modular Joint Design for a Performance Enhanced Humanoid Robot, in *Proc. IEEE Int. Conf. on Robotics and Automation*, Orlando, USA, pp.88-93 (2006)

[15] Y. Ogura, K. Shimomura, H. Kondo, A. Morishima, T. Okubo, S. Momoki, H. Lim and A. Takanishi, Human-like Walking with Knee Stretched, Heel-contact and Toe-off Motion by a Humanoid Robot, in *Proc. IEEE Int. Conf. Robotics and Automation*, pp.3976–3981 (2006)

[16] M. Hirose and K. Ogawa, Honda Humanoid Robots Development, *Phil. Trans. R. Soc. A*, 365, pp.11-19 (2007)

[17] F.R. Siasand and Y.F. Jr. Zheng, How Many Degrees-of-freedom Does a Biped Need?, in *Proc. IEEE Int. Workshop on Intelligent Robots and Systems*, Vol. 1, pp.297-302 (1990)

[18] C.L. Shih and W.A. Gruver, Control of a Biped Robot in the Double Support Phase. *IEEE Trans. on Systems, Man, and Cybernetics*, 22(4), 729-735 (1992)

[19] C.L. Shih, W.A. Gruver and T.T. Lee, Inverse Kinematics and Inverse Dynamics for Control of a Biped Walking Machine. *J. of Robotic Systems,* 10(4), 531-555 (1993)

[20] J. H. Park and K. D. Kim, Biped Robot Walking using Gravity-Compensated Inverted Pendulum Mode and Computed Torque Control, in *Proc. IEEE Int. Conf. on Robotics and Automation*, Leuven, Belgium, pp.3528-3533 (1998)

[21] Y. Ogura, H. Aikawa, H.O. Lim and A. Takanishi, Development of a Human-like Walking Robot Having Two 7-DOF Legs and a 2-DOF Waist, in *Proc. IEEE Int. Conf. Robotics and Automation*, pp.134-139 (2004)

[22] Y. Choi, D. Kim, Y. Oh and B.J. You, Posture/Walking Control for Humanoid Robot Based on Kinematic Resolution of CoM Jacobian With Embedded Motion. *IEEE Trans. on Robotics and Automation*, pp. 1285-1293 (2007)

[23] R. Featherstone and D. Orin, Robot Dynamics: Equations and Algorithms, in *Proc. IEEE Int. Conf. on Robotics and Automation*, Vol. 1, pp.826-834 (2000)

[24] J. Y. S. Luh, M. W. Walker and R. P. C. Paul, On-line Computational Scheme for Mechanical Manipulators, *ASME J. of Dyn. Sys. Meas. And Cont.* 102(2), 69-76 (1980)

[25] Y. Hurmuzlu and O. Nwokah, Humanoid Robots, Chapter 27 in *The Mechanical Systems Design Handbook: Modeling, Measurement, and Control*, CRC Press (2002)

[26] J.K. Salisbury, Active Stiffness Control of a Manipulator in Cartesian Coordinates, *19th Conf. on Decision and Control and Symp. on Adaptive Processes,* Albuquerque, USA, pp.95-100 (1980)

[27] C. L. Shih, Y. Z. Li, S. Churng, T. T. Lee and W. A. Gruver, Trajectory Synthesis And Physical Admissibility For A Biped Robot During The Single-Support Phase, in *Proc. IEEE Int. Conf. on Robotics and Automation*, Vol.3, pp.1646-1652 (1990)

[28] C. L. Shih, Y. Zhu and W. A. Gruver, Optimization of the Biped Robot Trajectory, in *Proc. IEEE Int. Conf. on Systems, Man, and Cybernetics*, Vol. 2, pp.899-903 (1991)

[29] S. Kajita, T. Yamaura and A. Kobayashi, Dynamic Walking Control Of A Biped Robot Along A Potential Energy Conserving Orbit, *IEEE Trans. on Robotics and Automation* 8(4), 431-438, (1992)

[30] S. Kajita, F. Kanehiro, K. Kaneko, K. Yokoi and H. Hirukawa, The 3D linear inverted pendulum mode: a simple modeling for a biped walking pattern generation, in *Proc. IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*, Vol. 1, pp.239-246 (2001)

[31] S. Kajita, F. Kanehiro, K. Kaneko, K. Fujiwara, K. Yokoi and H. Hirukawa, A Realtime Pattern Generator For Biped Walking, in *Proc. IEEE Int. Conf. on Robotics and Automation*, Vol. 1, pp.31- 37 (2002)

[32] S. Kajita, F. Kanehiro, K. Kaneko, K. Fujiwara, K. Yokoi and H. Hirukawa, Biped walking pattern generation by a simple three-dimensional inverted pendulum model, *Advanced Robotics* 17(2), 131-147 (2003)

[33] E. V. Cuevas, D. Zaldivar and R. Rojas, Bipedal Robot Description, Freie Universitat Berlin, Institut fur Informatik, Technical Report B-04-19 (2005)

[34] H. Qiang, K. Yokoi, S. Kajita, K. Kaneko, H. Arai, N. Koyachi And K. Tanie, Planning Walking Patterns for a Biped Robot. *IEEE Trans. on Robotics and Automation*, 17(3), 280-289 (2001)

[35] S. Kajita, F. Kanehiro, K. Kaneko, K. Fujiwara, K. Harada, K. Yokoi and H. Hirukawa, Biped Walking Pattern Generation by Using Preview Control of Zero-Moment Point, in *Proc. IEEE Int. Conf. on Robotics and Automation*, Vol. 2, pp.1620-1626 (2003)

[36] J. Park and Y. Youm, General ZMP Preview Control for Bipedal Walking, in *Proc. IEEE Int. Conf. on Robotics and Automation*, pp.2682-2687 (2007)

[37] K. Harada, S. Kajita, K. Kaneko and H. Hirukawa, An Analytical Method On Real-time Gait Planning for a Humanoid Robot, *in Proc. 4th IEEE/RAS Int. Conf. on Humanoid Robots*, 2, pp.640- 655 (2004)

[38] Y. Choi, B.J. You and S.R. Oh, On the Stability of Indirect ZMP Controller for Biped Robot Systems, *in Proc. IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*, 2, pp.1966-1971 (2004)

[39] T. Katayama, T. Ohki, T. Inoue and T. Kato, Design of an Optimal Controller for a Discrete-Time System Subject to Previewable Demand, *Int. J. of Control* 41(3), 677-699 (1985)

[40] R. A. Hess and Y. C. Jun, Generalized Predictive Control of Dynamic Systems, in *Proc. IEEE Int. Conf. on Systems, Man, and Cybernetics*, pp.844-849 (1988)

[41] T. Buschmann, S. Lohmeier, M. Bachmayer, H. Ulbrich and F. Pfeiffer, A Collocation Method for Real-Time Walking Pattern Generation, *in Proc. IEEE Int. Conf. on Humanoid Robotics* (2007)

[42] American Honda Motor Co., ASIMO Technical Information, Inc. Corporate Affairs & Communications (2003)